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AUTHOR	Heller, Joan I.; Reif, F.
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ABSTRACT

This study examined a proposed procedure for constructing theoretical descriptions of mechanics problems, in particular, to determine if the procedure led to explicit and correct descriptions of the motion and interaction of systems and if the resulting theoretical descriptions facilitated generation of correct constraint equations and, hence, correct solutions to the problems. Twenty-four paid volunteers (undergraduate physics students) were randomly assigned to one of three groups: (1) model group, induced to solve problems in accordance with the full version of the proposed procedure; (2) modified-model group, induced to work with a less complicated version; and (3) a comparison group which solved problems without any external guidance. Criteria used as measures of good problem-solving performance and major classes of errors were established and adequacy of solution was assessed with respect to these performance measures. Very explicit rules for constructing initial problem descriptions were found to lead reliably to explicit and correct descriptions of motion and interaction. In addition, these descriptions were found to facilitate achievement of correct solutions. Specific examples of typical difficulties subjects encountered during problem-solving are also discussed. (Author/JN)

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Cognitive Mechanisms Facilitating Human Problem Solving in Physics: Empirical Validation of a Prescriptive Model*

Joan I. Heller and F. Reif

University of California, Berkeley

INTRODUCTION

In the previous paper (Reif & Heller, 1982) we proposed a prescriptive model specifying procedures and knowledge structures required for good problem-solving performance in physics. We also outlined an experimental method for testing such a model. We now describe in more detail empirical work to test selected aspects of the proposed model.

In this experiment our primary interest was in evaluating the proposed procedure for constructing theoretical descriptions of mechanics problems. The particular questions we addressed were: (1) Does the procedure lead to explicit and correct descriptions of the motion and interaction of systems? (b) Do the resulting theoretical descriptions facilitate generation of correct constraint equations and, hence, correct solutions to the problems?

In the following sections we describe the method, results, and implications of research to validate these parts of the proposed prescriptive model.

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METHOD

The method we used to evaluate the proposed model was to compare the problem-solving performance of subjects working under different experimental conditions. The utility of the model was assessed by observing the performance of subjects who were induced to work in accordance with the model. These subjects were guided through solutions of problems by external control directions read to them by the experimenter according to a written script. By comparing the performance of subjects following these sets of directions with the performance of subjects working without external guidance, the facilitating effects of the proposed procedures could be determined.

Two different versions of the external control directions were developed, one of which (the "modified model" version) consisted essentially of a subset of the other (the "model" version). The modified version was developed to assess the importance of particular components of the model. If these components were necessary for good performance, the performance of subjects guided by directions that did not include these components would be expected to deteriorate in predictable ways compared to the performance of subjects guided by the full version.

The performance of three groups of subjects was compared in this study: a Model group (M) which was induced to solve mechanics problems in accordance with the full version of the proposed problem-solving procedure; a Modified-Model group (M*) which was induced to work in accordance with the less complete version of this procedure; and a Comparison group (C) which solved problems without any external guidance.

Subjects

The subjects were 24 paid volunteers who were undergraduate students currently enrolled in the second semester of an introductory physics course at the University of California at Berkeley. The physics principles and kinds of problems used in this study had been included in the first semester of this course. Hence it could be assumed that these subjects had learned this relevant knowledge just a few months before their participation in this research.

The subjects were selected randomly from those volunteers who had received a grade of B- or better in the first semester of the course. These subjects were then randomly assigned to the three groups, eight in each group.

Procedure

A pretest consisting of three mechanics problems was first administered individually to each subject. Subjects were asked to talk aloud about what they were thinking while solving the problems, and their verbalized statements were recorded with their permission. During this and subsequent sessions, the subjects were provided with a printed summary of relevant mechanics principles to which they could refer at any time. Because our interest was not in the subjects' knowledge about algebra or trigonometry, any apparent errors of these kinds were pointed out or corrected by the experimenter as they occurred.

Subjects in Groups M and M* then received brief training to familiarize them with the directions they were going to be asked to follow. This training consisted of a single practice run through the major steps of the problem-solving procedure.

Each subject then returned for one or two subsequent sessions during which three problems, approximately equivalent to the pretest problems, were administered individually. Groups M and M* were guided through the solution of these problems, while Group C again worked without external guidance. The subjects were asked to talk out loud and were tape recorded. The subjects' written work and verbalized comments comprised the data for this study.

Subjects working with external guidance were read the standard directions one step at a time. Each direction had to be implemented by the subject before the next one was read. If a step was not performed, the directions were repeated.

External Control Directions

Standard external control directions were developed for use with subjects in Groups M and M*. These directions provided very specific guidance through problem solutions but were problem-independent--i.e., the same directions were applicable to any mechanics problem that could be solved by application of Newton's second law. A summary and comparison of the kinds of knowledge included in the directions for the Model and Modified Model groups is provided in Table 1.

Insert Table 1 about here.

The external control directions specify procedures for accomplishing two major activities involved in problem solving: constructing an initial theoretical problem description, and synthesizing the problem solution by generating constraints (usually in the form of equations or inequalities).

The modified version is comprised essentially of a subset of the steps included in the full version. Whereas the full version includes descriptions of both the motion and interaction of systems, the modified version includes only a description of interaction. Furthermore, the full version includes a specific algorithm for constructing interaction descriptions; by contrast, the modified version only directs students to draw all forces acting on every system, without specifying in more detail how to enumerate them. Thus, the modified version corresponds roughly to the kind of suggestions a typical physics text provides--to draw "free-body" force diagrams of systems, without explicit rules for identifying or describing forces.

The full version also includes methods for checking that the motion and interaction of systems are correctly and conveniently described. One check involves the comparison of motion and interaction descriptions to ensure their consistency. This is only possible when both motion and interaction have been examined explicitly, as in the full version. A second check involves examination of interaction descriptions to ensure that constraints implied by Newton's third law have been considered--namely, that mutual forces (i.e., "actions" and "reactions") are described as equal in magnitude and opposite in direction.

Directions for synthesizing solutions are essentially identical in both versions. Of major interest here are the directions to choose explicitly a principle, a system, and a direction (or coordinate system) when applying Newton's second law to generate equations.

The way in which the differences between the full and modified versions were actually implemented is exemplified in Table 2 which contains excerpts from the scripts used to direct subjects through enumeration of forces acting on a chosen system.

Insert Table 2 about here.

Assessment Problems

Three approximately matching pairs of mechanics problems were selected from commonly used introductory physics texts (French, 1971; Resnick & Halliday, 1977; Symon, 1971). (The problems are listed in the Appendix.) These problems were reworded slightly for increased clarity. The pairs of problems were split into two sets, A and B. Half of the subjects received one set as a pretest and the other set during treatment sessions; the other half of the subjects received these sets in opposite order.

All of the problems used in the study could be solved by application of one fundamental motion principle, Newton's second law ($\underset{\sim}{F} = m\underset{\sim}{a}$). Two of the three pairs of problems (1A, 1B, 3A, 3B) required non-trivial force descriptions, i.e., several forces (including both contact and long-range forces) were involved. These problems were included to allow assessment of procedures for enumerating forces. The third pair of problems (2A and 2B) required non-trivial motion descriptions; they involved systems in circular motion, the analysis of which is frequently performed incorrectly by novices. These problems were included to allow assessment of procedures for describing motion.

Data Analysis

In order to assess the quality of students' problem-solving behavior, it was necessary to identify and define performance measures. The criteria used as measures of good performance, and major classes of errors, are provided in Table 3.

Insert Table 3 about here.

RESULTS

The adequacy of every solution was assessed with respect to the performance measures listed in Table 3. The data in Table 4 and Figure 1 show the mean number of each student's solutions (on the three problems solved during pretest or treatment sessions) that were correct on each of these measures.

Insert Table 4 about here.

Insert Figure 1 about here.

Performance on the pretest is summarized across all 24 students in Table 4 and Figure 1 to facilitate comparison with performance under the three treatment conditions. Statistically significant differences between groups in treatment sessions are indicated in the rightmost columns. There were no significant differences between groups on the pretests.

Sufficiency of the Model

The purpose of this research was to evaluate selected aspects of the proposed model of good problem-solving performance in mechanics. The major question of interest is whether the kinds of procedures proposed by the model are sufficient for producing successful solutions. If the kinds of knowledge included in the model are sufficient, students working in accordance with the model would be expected to perform well.

The performance of subjects in Group M, working under external control, indicates that the proposed procedures did lead to good performance. As indicated in Table 4 and Figure 1, these students performed nearly perfectly--all of their solutions contained every required equation, and all equations contained correct and complete information about motion and interaction. (The slightly lower incidence of correct final answers resulted from incorrect combination of equations on problem 2B; instead of performing a required vector addition, some students treated vectors as scalars.)

Adequacy of Performance Unguided by Model

The above finding indicates that performance in accordance with the model is excellent. However, it is possible to question whether performance might have been equally good without such intervention. These subjects received formal instruction in mechanics just one semester earlier, and might have the requisite knowledge for solving these fairly standard problems.

The subjects' performance on the pretest as well as the performance of Group C, as summarized in Table 4 and Figure 1, indicates that their prior knowledge was definitely not sufficient for the task. On the average, less than one of three pretest problems was solved correctly. Furthermore, the mean number of solutions containing a sufficient set of equations, or correct information about forces and motion, was always less than two. Group C's performance in the treatment sessions was virtually identical to the summarized pretest performance of all subjects.

These results indicate that the kind of knowledge students acquire as a result of ordinary instruction in an introductory physics course is not sufficient for the task of solving typical mechanics problems. It should be noted that the subjects in this study did have some knowledge of physics principles and definitions--they had enough knowledge to interpret and implement the external control directions. However, the additional procedures and knowledge provided by these directions was necessary to produce good problem-solving performance.

Necessity of Components of the Model

It has been shown that students working in accordance with the complete version of the model do perform well. A question that can then be raised is whether all of the components of the model are actually necessary. It may be that some of the procedures and knowledge structures are superfluous, and that performance might be equally good (and perhaps more efficient) without those parts. This question can be tested by comparing the performance of Groups M and M*.

Group M*, it will be recalled, worked in accordance with a subset of the knowledge provided to Group M (see Table 1). If the knowledge components, deleted from the directions used to guide Group M*, were in fact necessary, performance of this group should not be as adequate as that of Group M. In particular, since the major differences between the directions lay in the completeness and explicitness of guidance through initial problem description, it would be expected that motion and interaction analyses of Group M* should be inferior to those of Group M. In turn, the equations generated by subjects in Group M*, and final answers obtained, should be correct less often than those of Group M.

The data in Table 4 and Figure 1 reveal essentially this pattern of results. All results are statistically significant, except in the case of motion description, where the performance of Group M* was not significantly poorer than the perfect performance of Group M. It thus appears that the kinds of knowledge included in the model are both sufficient and necessary for achievement of good problem solving.

Qualitative Analysis

A closer examination of the subjects' performance provides insights into the ways in which the proposed model facilitates good performance. In this section we discuss some specific examples of typical difficulties subjects encounter during problem solving. For each such example, we indicate the particular components of the model that lead to good performance, i.e., prevent typical errors.

Missing forces. One of the most common errors made by the subjects was to omit mention of existing forces: 75% of the subjects omitted relevant forces in at least one of their pretest problem solutions. Performance on problem 3B exemplifies this difficulty. This problem involves two mutually sliding blocks (see Figure 2) where block B is pulled to the left with some force F_0. The blocks are connected by a string, with negligible mass, which passes over a pulley with negligible mass rotating with negligible friction. The solution of this problem requires identification of all forces on both blocks.

Insert Figure 2 about here.

Identification of all forces on block B presented particular difficulty for the subjects. In half of the pretest solutions of this problem, the friction force (f_{BA}) on B by block A was omitted; the tension force (T) and normal force (N_{BA}) on B by A were each omitted in 25% of the pretest solutions.

These errors were eliminated entirely by the model's algorithm for enumerating forces. The algorithm involves identifying all systems that touch the system being described, and additional knowledge is provided to remind subjects that the force exerted by a surface ordinarily has two components--the normal force and friction force. Identification of systems touching another system is trivial for the subjects. This procedure thus eliminates automatically the very common error of missing contact forces on a system.

It is interesting to note that subjects in Group M* also missed no forces on block B, although they were merely told to "indicate all the forces exerted on block B by all other systems" and then to check that they had identified all such forces. However, over all solutions, subjects in Group M never missed any forces, while those in Group M* did still do so (see Table 4 and Figure 1). Thus, the algorithm is far more reliable than the less specific directions provided to Group M*, although the latter produce better performance than that exhibited by students working without any guidance.

Wrong direction for force. A second very common error exhibited on the pretest was that of ascribing the wrong direction to a force. Half of the subjects made this error on at least one pretest solution. An example of this difficulty is provided in Figure 3.

Insert Figure 3 about here.

In this problem, blocks A and B are connected by a string of negligible mass which passes over a pulley of negligible mass and negligible friction. It is specified that block C, which sits on top of block A, remains at rest relative to A (i.e., does not slide off).

In order to solve problem 1A, the forces on block C must be identified and described correctly. However, on 83% of the pretest solutions of this problem, the friction force on C by A was described as acting on C to the left when in fact it is exerted to the right. (If the friction were exerted to the left, C would surely slide off A! It is only the friction force of A on C which moves C to the right.)

This error appears to be the result of the (verbalized) rule the subjects use to determine the direction of a friction force: "friction opposes the motion (of C)." This rule is too general--it only leads to correct force description under certain conditions. A more specific rule is required, "friction opposes relative motion of the contact points",-- i.e., in this case, friction opposes the motion of C relative to A. Subjects in Group M, who used this rule, never erred in ascribing the correct direction to the friction force; by contrast, subjects in Groups M* and C continued to make such errors at the same rate as all subjects on the pretest solutions.

The model provides not only explicit rules for correctly describing forces, but includes also checks to ensure that forces have been described properly. One such check requires that the descriptions of the motion and interaction of each system be consistent. In order to perform this check, both the motion and interaction must have been described— as required by the model. For example, for the problem illustrated in Figure 3a, the description procedure generates both a motion diagram and force diagram, as illustrated in Figures 3b and c.

Newton's second law ($\underset{\sim}{F} = m\underset{\sim}{a}$) implies that the motion and interaction of a system are related. Hence it provides the following check of consistency of motion and interaction, included in Group M's directions: "In your diagrams, are the forces on the selected particle such that, with proper magnitudes, their vector sum can have the same direction as the particle's acceleration? If not, there is something wrong." If the friction force on C were described to the left, as in Figure 3b, and the acceleration were described to the right, as in Figure 3c, this check would reveal that an error was made in generating one of the descriptions. Since it is easy for subjects to determine the direction of block C's acceleration in this problem, this consistency check provides a very reliable means for blocking the common error of incorrectly describing the direction of the friction force on C.

Furthermore, this comparison of motion and force diagrams appeared to provide the subjects with an extremely powerful graphic demonstration of the meaning of Newton's second law. Many of the subjects in Group M spontaneously reacted to this comparison with comments indicating a new understanding of the implications of $\underset{\sim}{F} = m\underset{\sim}{a}$, e.g., "Oh! That's neat! I hadn't thought about it that way before." The potential of this kind of procedure for enhancing students' understanding of physics principles may be a fruitful area for investigation.

Another check on initial theoretical descriptions consisted of determining whether mutual forces (i.e., "actions" and "reactions") were correctly described in accordance with Newton's third law. Subjects in Group M were directed to do the following: "Check to make sure that all action-reaction pairs of forces are described as equal in magnitude and opposite in direction. For example, if systems A and B interact, the force on A by B in your force diagram of A should be opposite in direction but should have the same magnitude as the force on B by A in your diagram of B." Thus, subjects would be led to compare their force diagram of C (Figure 3b) with their force diagram of A (see Figure 4).

Insert Figure 4 about here.

According to the constraint implied by Newton's third law, the friction force on C by A must be opposite in direction to the friction force on A by C. The discovery that they both point in the same direction would serve as a strong indicator that an error has been made.

CONCLUSIONS

This research was conducted to validate selected aspects of a proposed prescriptive model of good problem-solving performance in mechanics. In particular, very explicit rules for constructing initial problem descriptions were found to lead reliably to explicit and correct descriptions of motion and interaction. Furthermore, these descriptions were found to facilitate achievement of correct solutions. These findings suggest that successful problem solving in the domain of mechanics is facilitated by initial problem descriptions containing the following elements:

* An explicit description of both motion and interaction of systems.
* Special knowledge about the properties of such forces.
* Checks on descriptions based on consistency with physical laws.

It was found that, even after receiving traditional formal instruction in mechanics, students are still quite deficient in skills required for the solution of fairly routine problems. Additional knowledge of the type included in the proposed model is necessary for achieving good performance. It has been demonstrated that it is possible to explicate those specific kinds of knowledge and procedures which, if utilized, can improve performance. These aspects of problem-solving skill are typically not made explicit in physics courses. However, as we have shown, they can dramatically improve students' performance.

While we have focused here on knowledge for describing problems, our theoretical ideas encompass other aspects of problem solving, including planning and synthesis of solutions by the generation of constraint equations. In future work we hope to explore and refine these additional areas, as well as to generalize beyond the domain of mechanics.

In this study we have used an experimental technique which involves observation of subjects working under external control in accordance with different versions of a proposed model. This method allows the researcher to explore in some detail the effectiveness of prescribed knowledge and to manipulate experimentally various parameters of the model. The technique may be broadly useful for exploring the utility of procedures and knowledge structures in a wide variety of domains.

This work has been motivated by the assumption that the design of effective instruction in scientific problem solving is only possible once reliable problem-solving methods have been specified. Toward this end, we have been developing and testing methods leading to effective problem solving. Up to now, our work has not been directly aimed at developing instruction in problem solving, but has focused on an important prerequisite to research on instruction. After having demonstrated the utility of certain aspects of a prescriptive model of good problem solving, our next task will then be to begin exploring ways to teach this knowledge so that students both internalize it and use it spontaneously. Such work on effective modes of teaching scientific problem solving is being planned and will be reported in future publications.

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Table 1

Major Components of External Control Directions for Model (M) and Modified Model (M*)

Components common to both versions	Additional components in model M only.
Interaction Description	
Direction to draw separate force diagrams indicating all forces exerted on each system by all other systems.	Specific algorithm for enumerating forces. Special knowledge about properties of interactions (e.g., explicit rules for determining directions of forces).
Motion Description	
Explicit mention of motion in context of constraint generation.	Direction to draw separate motion diagrams indicating position, velocity, and acceleration of each system. Special knowledge about motion (e.g., explicit information about components of acceleration of systems with circular motion).
Checks on Descriptions	
Reminder to choose useful symbols. Check that all given information has been used.	Check for consistency of motion and interaction descriptions. Check that mutual forces are described correctly (equal in magnitude, opposite in direction).
Synthesis of Solution	
Explication of kinds of decisions to be made during application of motion principles (choice of principle, system, direction). Assessment of current problem state.	

Table 2

Examples of External Control Directions
for Constructing Interaction Descriptions

Model M:

E: Let's now draw diagrams describing the forces on each system of interest. Which system...do you wish to consider first/next?

S: (Names a system "S".)

E: First name each system that touches S, including those that exert applied forces. As you identify each system, indicate all external contact forces exerted on S by that system.

S: (Names systems and indicates contact forces.)

**IF NAMED SYSTEM INTERACTS BY SURFACE CONTACT:

E: Remember, the force exerted by a surface ordinarily, although not always, has two components, the normal force and friction. Check to be sure whether both components exist in this case.

Also, remember that the normal force is perpendicular to, and directed away from, the surface exerting it. The friction force opposes the relative motion of the contact points; it opposes the motion of S relative to (interacting system).

E: Now name all external systems that directly interact with S without touching it or through any other physical contact. Then indicate the long-range forces exerted on S by each such system.

S: (Names systems and indicates long-range forces.)

E. Are there any other systems touching S?

S: (Indicates no others or names additional system(s) and indicates contact force(s).)

E: Are there any other systems directly interacting with S by long-range forces?

S: (Indicates no others or names additional system(s) and indicates long-range force(s).)

E: If not, you are finished describing all forces on S. DO NOT ADD ANY OTHERS.

Table 2 (cont'd)

Modified Model M*:

E: Let's now draw diagrams describing the forces on each system of interest. Which system...do you wish to consider first/next?

S: (Names a system "S".)

E: Draw a force diagram indicating all the forces exerted on S by all other systems.

S: (Draws a diagram.)

E: Are there any other forces exerted on S by any other systems?

S: (Indicates no others or draws additional forces.)

Table 3

Performance Measures and Error Types

Performance Measure	Major Error Types
1. Correctness of final answer: Was correct answer obtained?	Incorrect (or no) final answer.
2. Adequacy of constraint equations: Were the number and kinds of equations generated sufficient to determine a solution, and were all equations correctly instantiated?	Missing required equation. Incorrect information contained in equation. Meaningless equation (e.g., confused systems).
3. Adequacy of interaction information utilized: Were all required forces included in equations, and were directions and magnitudes of those forces correctly indicated?	Missing force(s) in equation. Wrong direction of a force. Wrong magnitude of a force.
4. Adequacy of motion information utilized: Was information about the magnitude and direction of each system's acceleration correctly included in equations?	Wrong direction of acceleration. Wrong magnitude of acceleration.

Table 4

Mean Number of Solutions with Correct Performance on Specified Measures

Performance measures	Pretest[a]	Treatment[b] M	M*	C	Statistical differences[c] M>M*	M*>C	M>C
Correct motion information	1.83	3.00	2.63	1.63	-	-	*
Correct force information	1.33	3.00	2.00	1.38	**	-	**
Sufficient and correct set of equations	.83	2.88	1.63	.75	**	-	**
Correct final answer	.79	2.75	1.38	.63	*	-	**

Note: Maximum score = 3.00

[a]n = 24

[b]n = 8 per group

[c]Kruskal-Wallis Test results: *p<.01; **p<.005

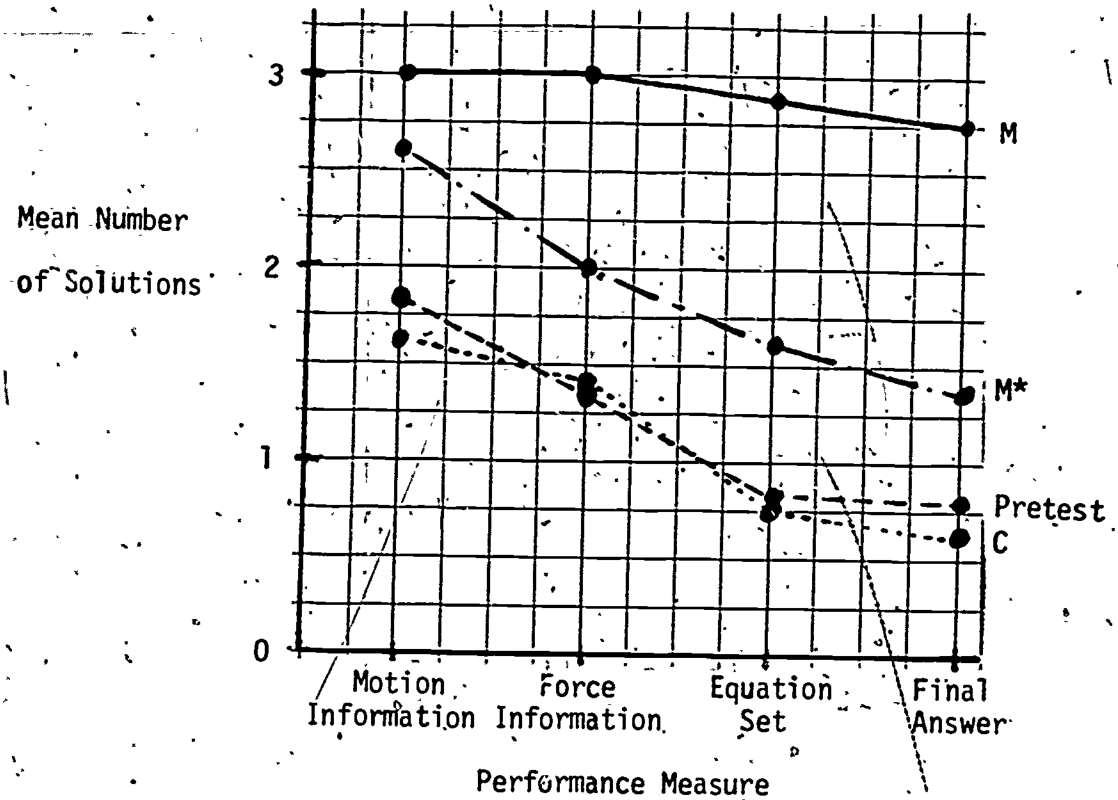


Figure 1. Graph of mean number of solutions with correct performance on indicated measures (for all subjects on pretest, and subjects in Groups M, M*, and C in treatment sessions).

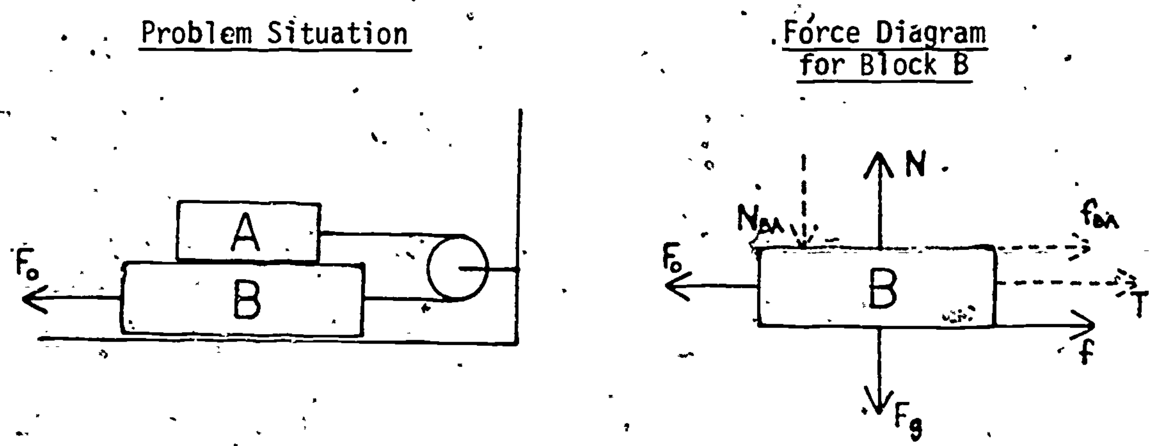


Figure 2. Problem situation in problem 3B and force diagram for block B. (Forces never missed by subjects in this study appear as solid arrows; forces frequently missed appear as dotted arrows.)

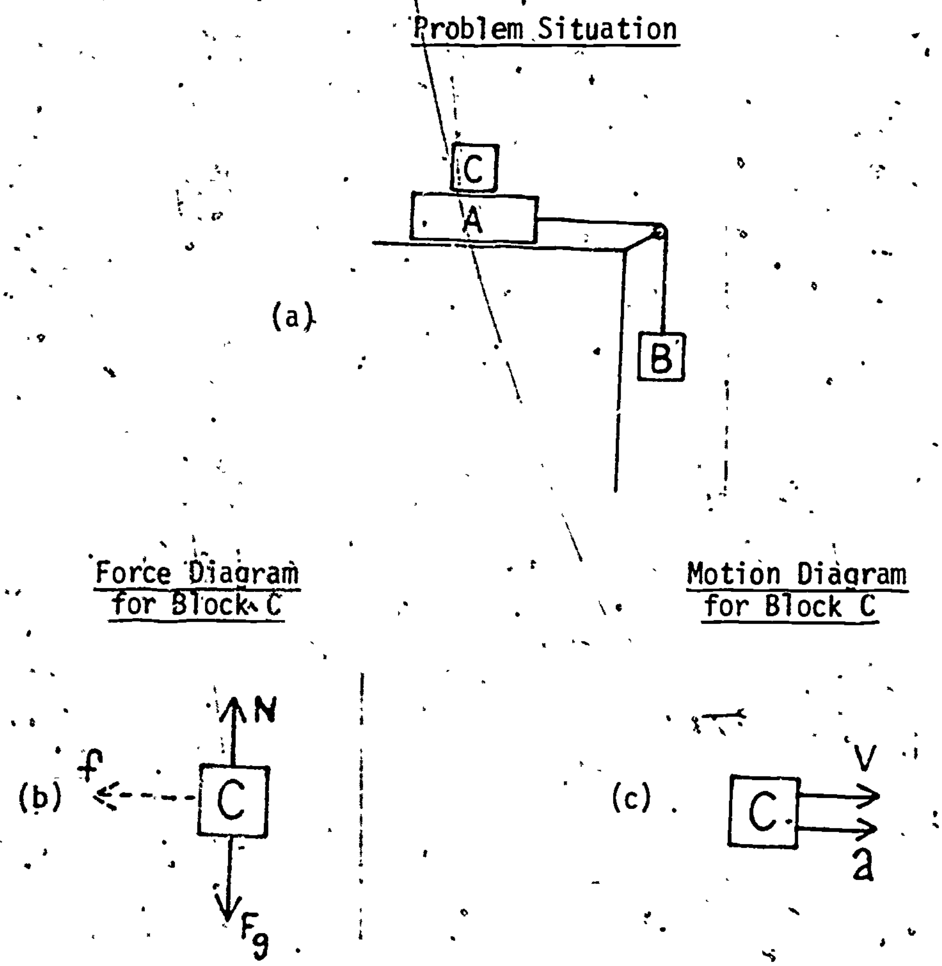


Figure 3. Problem situation in problem 1A, with force and motion diagrams for block C. (The dotted arrow indicates the wrong direction commonly ascribed to the friction force on C.)

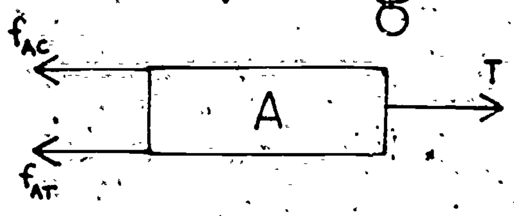


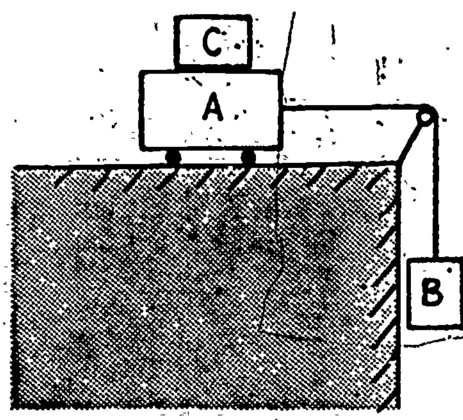
Figure 4. Horizontal forces on block A in problem 1A.

APPENDIX

Block lying on moving cart

The diagram shows a cart A (of mass 2m) free to move without friction along a horizontal table. This cart is attached by a light string, which passes over a massless frictionless pulley, to a block B (of mass m_B) suspended from the other end of the string. A block C (of mass m) lies on top of cart A. The coefficient of static friction between A and C is μ.

What is the maximum value of m_B for which block C will remain on the cart without sliding?



Specified information

cart A:	mass 2m coefficient of static friction between A and C = μ	string:	massless
		pulley:	massless frictionless
block B:	mass m_B	table:	horizontal frictionless
block C:	mass m does not slide off A coefficient of static friction between A and C = μ		

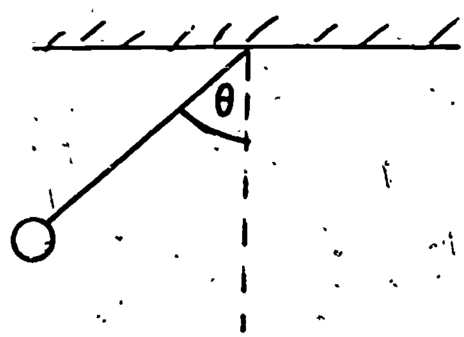
Goal

maximum m_B = ?

Swinging pendulum

A pendulum bob, of mass m, swings in a vertical plane at the end of a massless string fastened to the ceiling. At the highest point of its swing, the pendulum is in the position shown in the figure, with the string at an angle θ from the vertical.

What is the magnitude of the tension force exerted on the bob by the string at this instant?



Specified information

pendulum bob:	mass m at angle θ from vertical at highest point of swing tension force $\underset{\sim}{T}$ exerted on bob by string
string:	massless

Goal

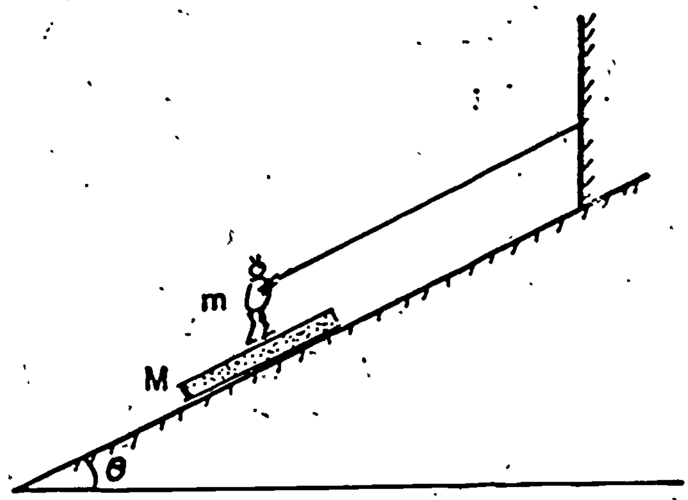
$T = ?$

Man standing on sliding board

A man, of mass m, stands on a board, of mass M, which he previously placed on a mud-covered hilly surface making an angle θ with the horizontal. The man holds on to a rope (of negligible mass and parallel to the surface of the hill) whose other end is fastened to a wall at the top of the hill. (See the diagram.)

The man finds, to his dismay, that the board beneath him starts sliding down the hill. The coefficient of sliding friction between the man's shoes and the board is μ_1, and the coefficient of sliding friction between the board and the surface of the hill is μ_2.

What is the magnitude of the acceleration, $\underset{\sim}{a}_B$, with which the board beneath the man slides down the hill while the man, holding on to the rope, remains at rest relative to the ground?



Specified information

board:	mass M acceleration $\underset{\sim}{a}_B$ coefficient of sliding friction between board and man, μ_1 coefficient of sliding friction between board and hill, μ_2
man:	mass m at rest relative to ground coefficient of sliding friction between man and board, μ_1
rope:	negligible mass parallel to hill fastened to wall at top of hill
hill:	at angle θ with horizontal coefficient of sliding friction between hill and board, μ_2

Goal

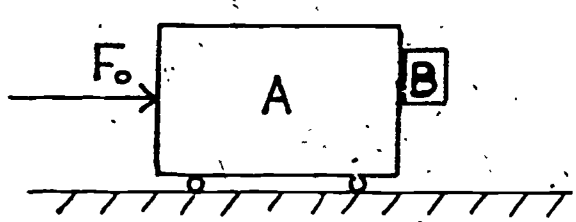
$a_B = ?$

Problem 1B

Block on side of moving cart

The diagram shows a cart A, of mass m_A, which moves with negligible friction along a horizontal floor when it is pushed to the right by an applied force of magnitude F_o. A small block B, of mass m_B, is in contact with the right vertical side of the cart. The coefficient of static friction between the block and the side of the cart has a value μ.

How large must be the magnitude F_o of the applied force so that the block remains at rest relative to the cart, without slipping down?



Specified information

cart A:	mass m_A applied force F_o on A to right coefficient of static friction between A and B = μ
block B:	mass m_B coefficient of static friction between A and B = μ does not slip down
floor:	horizontal frictionless

Goal

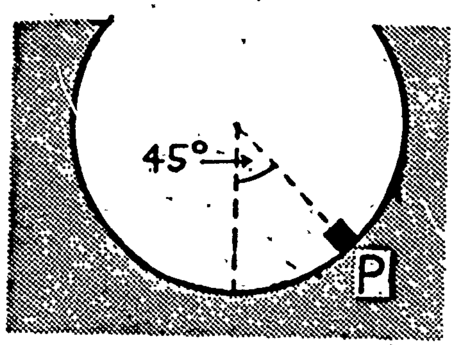
$F_o = ?$

Problem 2B

Object sliding along a circular track

An object of mass m slides along a frictionless circular track. When the object passes the point P in the figure below, the magnitude of the force exerted on the object by the track is $3mg/\sqrt{2}$.

What is the magnitude of the object's acceleration at that instant? (Use the values: $\sin 45^o = \cos 45^o = 1/\sqrt{2}$.)



Specified information

object:	mass m acceleration a at point P, at 45^o angle from vertical force of $3mg/\sqrt{2}$ exerted on object by track
track:	frictionless circular

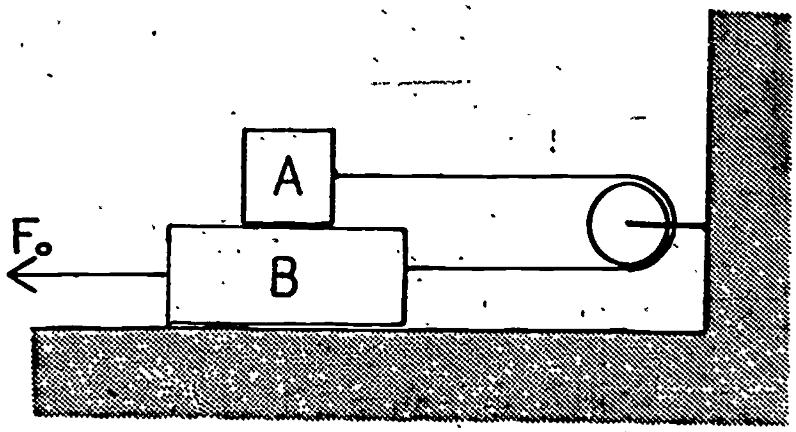
Goal

a = ?

Force on mutually sliding blocks

Two blocks, A and B, are connected by a light flexible string passing around a frictionless pulley of negligible mass. Block A has a mass m_A and block B has a mass m_B. The coefficient of sliding friction between the two blocks, and also between block B and the horizontal table below it, has a value μ.

What is the magnitude F_o of the force necessary to pull block B to the left at constant speed?



Specified information

block A: mass m_A
coefficient of friction between A and B, μ

block B: mass m_B
speed constant
coefficient of friction between A and B, μ
coefficient of friction between B and table, μ
applied force F_o on B to left

string: massless
flexible

pulley: massless
frictionless

table: horizontal
coefficient of friction between B and table, μ

Goal

$F_o = ?$